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# Static Coverage Path Planning for UAVs with Conical Field of View When Monitoring Rectangular Ground Areas

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**Abstract**— Finding a suitable UAV path to cover a given ground area is of paramount importance when accomplishing monitoring or mapping missions. In order to obtain an appropriate UAV route to inspect a rectangular ground area using a sensor with a conical field of view, this paper analyzes three classic trajectories, namely scan, rectangular spiral and Hilbert paths. Using carefully chosen metrics we conclude that the scan path is the right solution to this static coverage planning problem.

**Keywords**— *unmanned aerial vehicle (UAV); quadcopter; coverage path planning; mapping.*

## I. INTRODUCTION

The Unmanned Aerial Vehicles (UAVs) are intensively used in monitoring and mapping applications for both civilian and military purposes in a wide range of domains ranging from photogrammetry [1] and surveillance [2][3] to crop field monitoring [4] and disaster management [5]. In many of these applications the use of UAVs is directly related to the Coverage Path Planning (CPP) problem where the goal is to determine the path that guarantees that every point in a given environment was inspected [6].

There is a significant body of research addressing the CPP problem in both 2D or 3D environments [6][7]. To achieve a provable coverage guarantee, most of the coverage path planning algorithms relies on cellular decomposition of the target space [8]. This may be seen as a solution in the case of UAV sensors with a rectangular pyramidal field of view but for sensors with conical field of view such decomposition is unrealistic.

In an attempt to fill this research gap, current paper reduces the 3D path planning of the UAV to a 2D problem, by considering the trace of a circular shape with its center being the UAV's ground plane projection. Three classic static trajectories (i.e. scan, rectangular spiral and Hilbert paths) are considered and analyzed in this respect, concluding that the most suitable UAV route is given by the scan path.

The rest of the paper is structured as follows. Section 2 describes the static path planning problem. In Section 3, the three mentioned classic paths are presented, while Section 4 investigates these paths by considering a properly chosen set of metrics and also the specificity of the problem (the paths

are meant to be pursued by a quadrotor UAV having a sensor with a conical field of view). Finally, Section 5 concludes the paper.

## II. PROBLEM FORMULATION

Let us consider a quadcopter UAV used to scrutinize a given rectangular ground area using a sensor with a conical field of view described by the angle  $\alpha$  as presented in Figure 1.

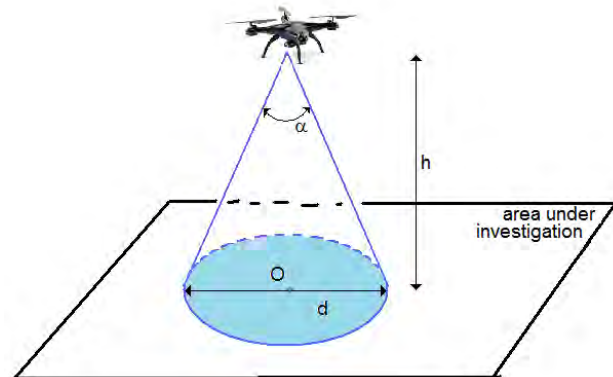


Figure 1 UAV monitoring an area with conical field of view

The sensor always points down, being mounted on a gimbal structure to compensate for the movements of the UAV. When evolving at height  $h$ , at each time instant the covered ground area is a circle of diameter  $d$ .

The goal of the problem is to find a suitable path, described by a series of waypoints, that satisfies the following criteria [9]: (i) the rectangular ground area is fully covered, meaning that every point of the area was visited at least one time; (ii) there is no repetition in the path, meaning that any point of the area is visited only once; (iii) the path is a sequence of simple motions (e.g. go-forward, rotate).

We assume that: a) the initial point of the UAV is right above a corner of the area under investigation, at a given  $h$  altitude; b) the ground area under investigation is flat; c) there are no obstacles to be avoided inside the flying zone or to obstruct the UAV sensor field of view.

This 3D path planning problem may be efficiently solved by reducing it to a 2D problem: instead of considering the movement of the UAV we may consider the movement of the UAV projection in the ground plane. In order to find a suitable solution we will pursue the following two steps: A) we will examine three typical path shapes {i.e. scan, rectangular spiral and Hilbert paths) when covering a square shape by considering specially chosen metrics and also the specificity of our problem (the paths will be pursued by a quadrotor UAV having a sensor with a conical field of view); and B) we will reconfigure the most suitable solution obtained in the previous step to cover a rectangular ground area.

### III. STATIC PATHS

In order to select a suitable trajectory for the quadrotor UAV that may solve our problem we will briefly analyze three basic and intensively used static path planning schemes (scan, rectangular spiral and Hilbert) in filling a square of side length  $L$ . These simple paths are all sequences of line segments joined end-to-end at right angles.

#### A. Scan

Scan, also known as back-and-forth, is a simple static path planning method where the UAV's ground projection traverses the area under investigation along a single specified dimension (either along x or y axis), while the distance between two consecutive segments along the specified dimension defines the trajectory resolution ( $R$ ). We may compute the total length of this path using [10]:

$$D_{Scan} = \left(\frac{L}{R} + 2\right)L \quad (1)$$

Figure 2 presents a scan path along y-axis for  $L=7$  and  $R=1$ .

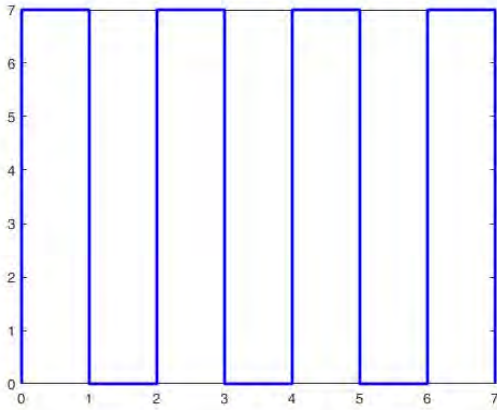


Figure 2 Scan path

It is important to note that the scan path can be simply adapted for convex polygonal areas [11].

#### B. Rectangular Spiral

By following a rectangular spiral shape, the area is covered along two dimensions (both x and y axes), either from the center of the area with increasing line segment lengths or from outside with decreasing line segments. The total length of the spiral path can be computed by:

$$D_{Spiral} = \left(\frac{L}{R} + 2\right)L \quad (2)$$

obtaining the same result as for scan path.

Figure 3 shows the rectangular path covering a square of side  $L=7$  with a resolution of  $R=1$ .

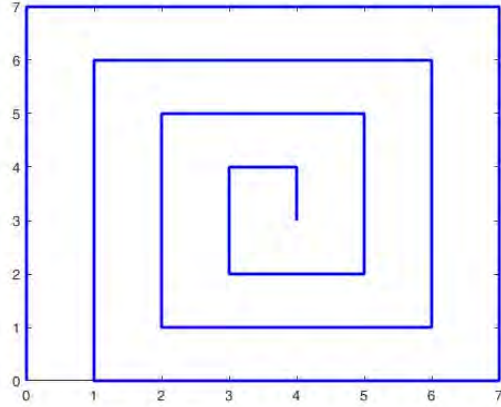


Figure 3 Rectangular spiral path

It is worth mentioning that the rectangular spiral path can be simply adapted for convex polygonal areas [11].

#### C. Hilbert

The Hilbert space-filling curve of order  $n$  is a continuous fractal path that traverses all the  $2^n \times 2^n$  cells in which the square area is divided without crossing itself. The Hilbert path may be constructed using simple iterative rules starting from a U shape element when  $n=1$  and applying a combined scaling-translation-rotation operation to derive more and more complex patterns with increasing density [12][13]. The length of Hilbert path inside a square of side  $L=2^n$  and resolution  $R$  is [14]:

$$D_{Hilbert} = \frac{4^n}{R} - R = \frac{L^2}{R} - R \quad (3)$$

In Figure 4, the Hilbert path of order  $n=3$  is presented.

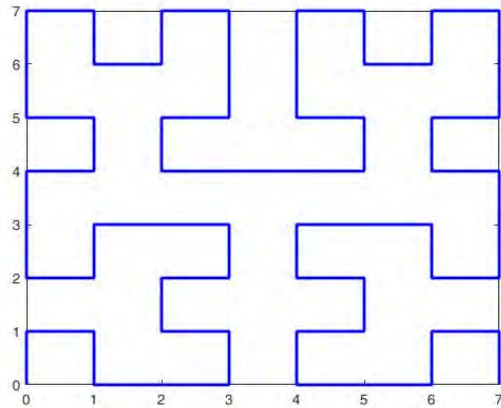


Figure 4 Hilbert path for  $n=3$

#### D. Evaluation of the space-filling paths

It worth mentioning that the Hilbert order  $n$  may take only positive integer values, thus complete area coverage cannot be obtained for every  $(L, R)$  pair. For this reason, to obtain a fair

comparison between the three paths we selected a square area with side  $L=7$  and a scanning resolution  $R=1$  (these values were already used for plotting the Figures 2-4).

In the first stage of our evaluation we will consider four criteria when comparing the three paths (i.e. scan, rectangle spiral and Hilbert):

- Path length: the space-filling must be done with a reduce trajectory length;
- Number of direction changes: this parameter must be as low as possible. When the quadcopter UAV performs a turning maneuver, it almost stops [15]. Thus, a lesser number of turns provides a lesser time for pursuing the path;
- Shape of the covered area: a more general shape is desirable;
- Resolution of the path: in an ideal case, the distance between two parallel path line segments may take any real positive value.

The results are presented in Table 1.

TABLE I. COMPARISON OF THE THREE CONSIDERED PATHS

	Scan	Rectangle spiral	Hilbert
<b>Path length</b>	63	63	63
<b>Direction changes</b>	14	14	50
<b>Covered area</b>	polygon	polygon	square
<b>Resolution</b>	$R \in \mathbb{R}_{>0}$	$R \in \mathbb{R}_{>0}$	$R \in \{L \cdot 2^{-n}   n \in \mathbb{N}\}$

Analyzing this table we notice that:

- Hilbert path is the most inappropriate to be used. The number of direction changes is by far the highest, the covered area shape has a very particular form (i.e. a square) and the resolution of the path cannot take any real positive value since it is related with  $L$  and  $n$  by the formula:

$$R = \frac{L}{2^n}, n = 1, 2, \dots \quad (4)$$

- Scan and rectangle spiral paths seem to be suitable candidates having the same values for all the four chosen criteria.

A final decision between the three considered paths will be made based on the conical shape of the UAV's sensor field of view and also on the UAV flight characteristics in the following paragraphs.

A specific aspect that must be taken into consideration when comparing the three trajectories is directly related to the conical shape of the UAV sensor's field of view. The trace of a circular shape when dragged along a path made of linear segments is characterized by rounded (not sharp) corners, which inevitably brings to coverage holes. In this respect, a suggestive example is given in Figure 5, where the center of a circle with diameter  $d=1$  follows a simple path described by the following sequence of waypoints:  $(1,0)$ ,  $(1,2)$ ,  $(3,2)$ ,  $(3,0)$ ,  $(5,0)$ .

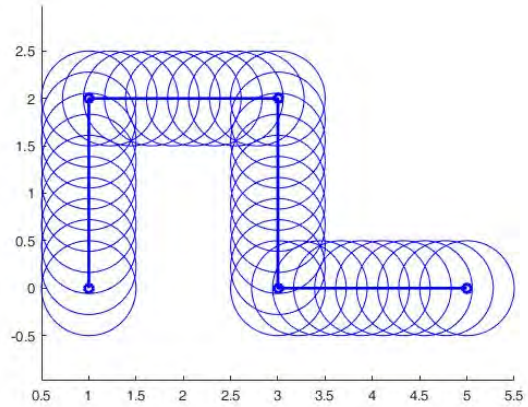


Figure 5 Trace of a circle dragged along a segmented path

This geometry-related issue substantially affects the coverage of the three considered path shapes. In Figures 6-8 we present the coverage for each of the three paths presented in Figures 2-4 when using a conical field of view. For this, we simulated in Matlab 2018b the trace when dragging the circular shape coverage (with the diameter  $d$  being equal with the resolution  $R$ ) along the paths.

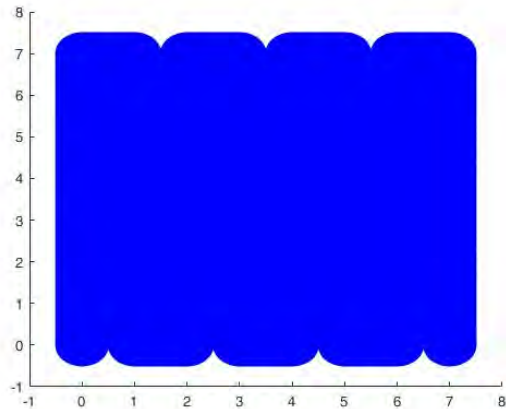


Figure 6. Coverage area with scan

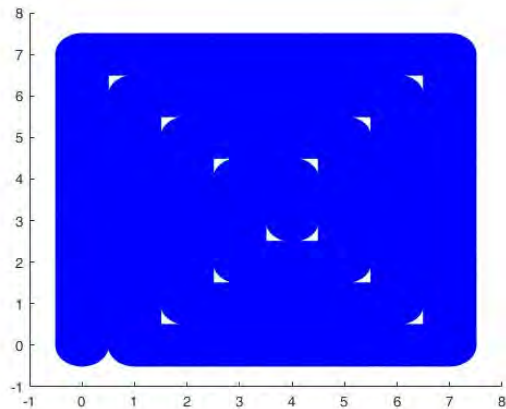


Figure 7. Coverage area with rectangular spiral

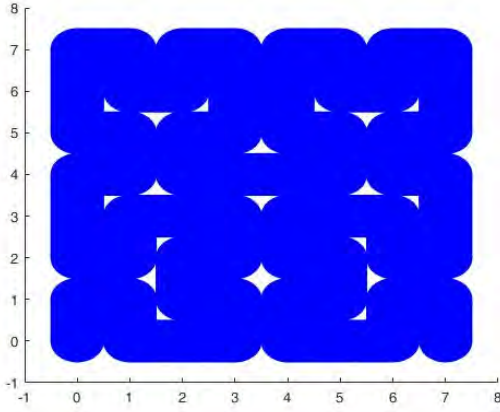


Figure 7. Coverage area with Hilbert

As may be noticed, every time a path is changing its direction an uncovered spot is obtained. For a complete coverage of the area, these small regions need also to be covered. Due to the position of these blind spots, the scan path is the best fitted candidate path for completing the area under investigation (i.e. the square having the vertexes with the coordinates (0,0), (0,7), (7,0) and (7,7)), so no additional corrective path is needed.

Another aspect that has to be considered when evaluating the paths is related to the UAV flight characteristics. When rotating the quadcopter almost stops and when flying horizontally between two successive waypoints the quadcopter speed-time profile may be approximated with a trapezoid [16]. Due to this reason, the scan path is more advantageous, the waypoints being linked either by very long line segments where UAV may travel most of time at full speed gaining time or very short line segments where the UAV accomplishes two successive direction changes without losing much time.

In conclusion, based on the four criteria depicted in Table 1 and also by analyzing the influence of the conical shape of the UAV's sensor field of view and the UAV flight characteristics, scan path is the most suitable trajectory for solving the given problem.

Having the results for a square ground area we may simply extend the results to a rectangular ground area having the sides  $L_1$  and  $L_2$ , the scan path length being computed using:

$$D = \left(\frac{L_1}{R} + 1\right) L_2 + L_1 \quad (5)$$

while the number of turns is:

$$N = 2L^* + 1 \quad (6)$$

where  $L^*=L_1$  if the rectangle area is scan along the side having the length  $L_2$  and  $L^*=L_2$  if the area is scan along the rectangle side having the length  $L_1$ . Inspecting (6) we observe that in order to obtain the minimum number of direction changes the scanning must be done along the major axis of the rectangle. This strategy is also in line with the need to have long path segments where the UAV average speed is increased.

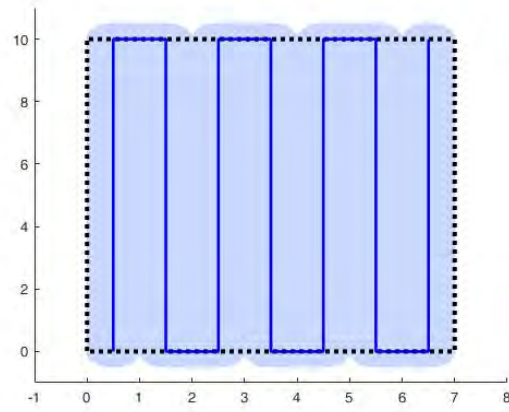


Figure 9. Optimal scan path for covering a rectangular area

Figure 9 presents the ground projection of the optimal scan path (solid line) needed to fully cover the rectangular area (dotted line) having the vertexes (0,0), (10,0), (10,7), (0,7) with a resolution  $R=d=1$ , while in gray we marked the actually covered area. For this, the quadcopter UAV will start its scan path in  $(R/2,0)$ :

#### IV. CONCLUSIONS

In this paper we investigated the static coverage path planning problem in the case of a quadcopter UAV equipped with a sensor having a conical field of view. After reducing the given 3D problem to 2D, we studied three classic path candidates (i.e. scan, rectangular spiral and Hilbert paths). Using properly chosen metrics and analyzing the influence of the conical shape of UAV's sensor field of view and also of the quadcopter flight characteristics, we concluded that the most appropriate UAV trajectory for covering a rectangular ground area is given by scan path.

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